Multidimensional Item Response Theory

What have we learned thus far...

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First of all

- This presentation is largely based on:
  - M.D. RECKASE (2009) *Multidimensional Item Response Theory: Statistics for Social and Behavioral Sciences*. New York, NY: Springer.

Summary

- Introduction to IRT
  - Advantages
  - Limitations
- MIRT
  - Solutions
  - Empirical data
  - Models
    - Compensatory
    - Partially compensatory
  - Item Information Surface.
  - Advantages of using MIRT

Item Response Theory

- Various advantages:
  - Sample independent parameters and estimates
  - Increased precision in estimates
  - Standard metric of estimates
    - Between subjects
    - Between tests
  - Simpler composition of tests for specific audiences
    - Item banks

  - Computerised Adaptive Testing

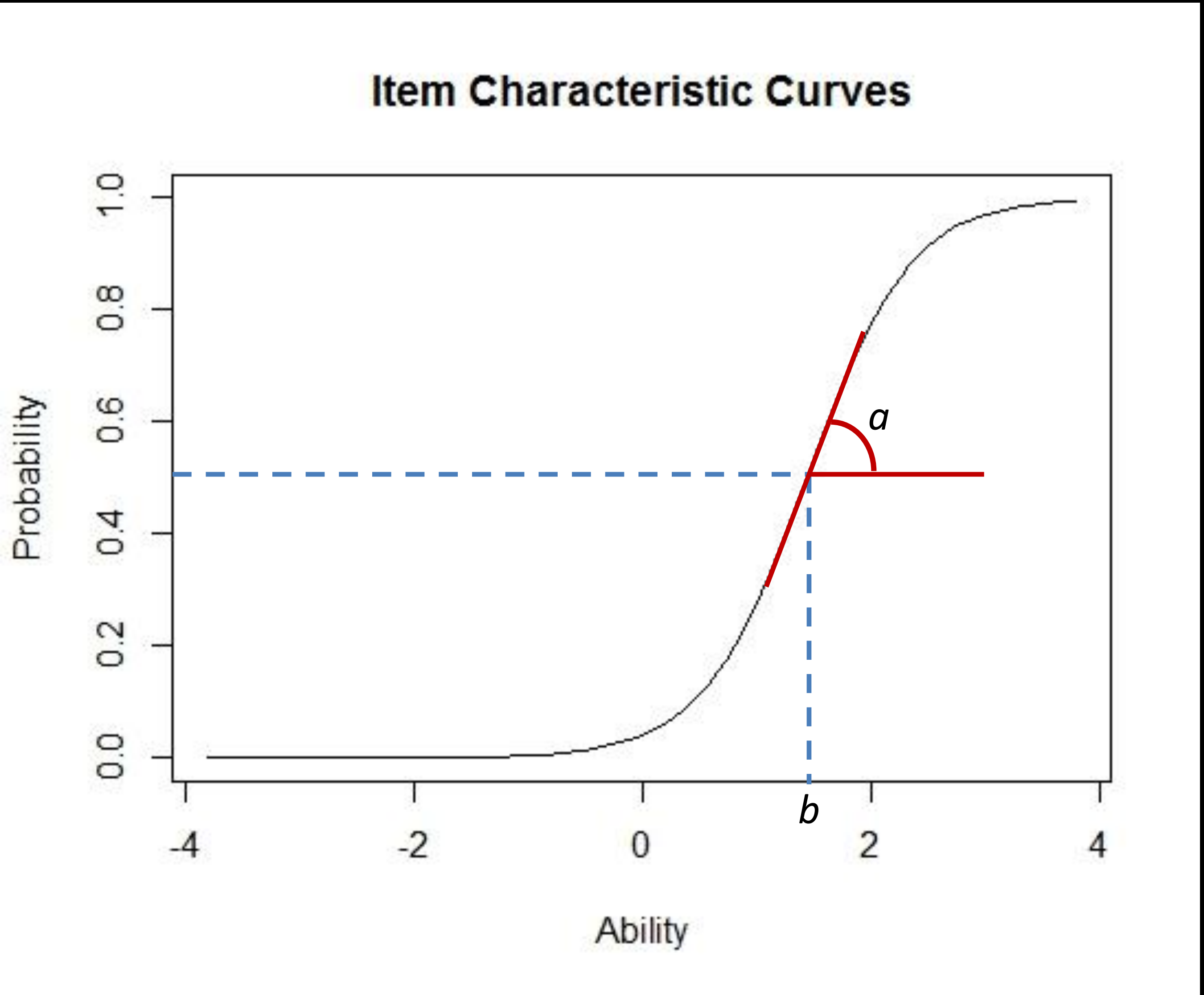
Item Response Theory

- Advancements from IRT models
  - More information on item level
    - **Proportion correct (p – value):** a single value that describes the item.
    - **Item difficulty:** summarized as *b* at 50% probability of scoring correctly, but provides information throughout the ability spectrum.

Item Response Theory

- Advancements from IRT models
  - More information on item level
    - **Discrimination in CTT :** biserial (or point-biserial) correlation between item score and total score.
    - **Item discrimination:** slope of the tangent line to ICC summarized as *a* at its maximum point.
      - Obtainable at any θ value.

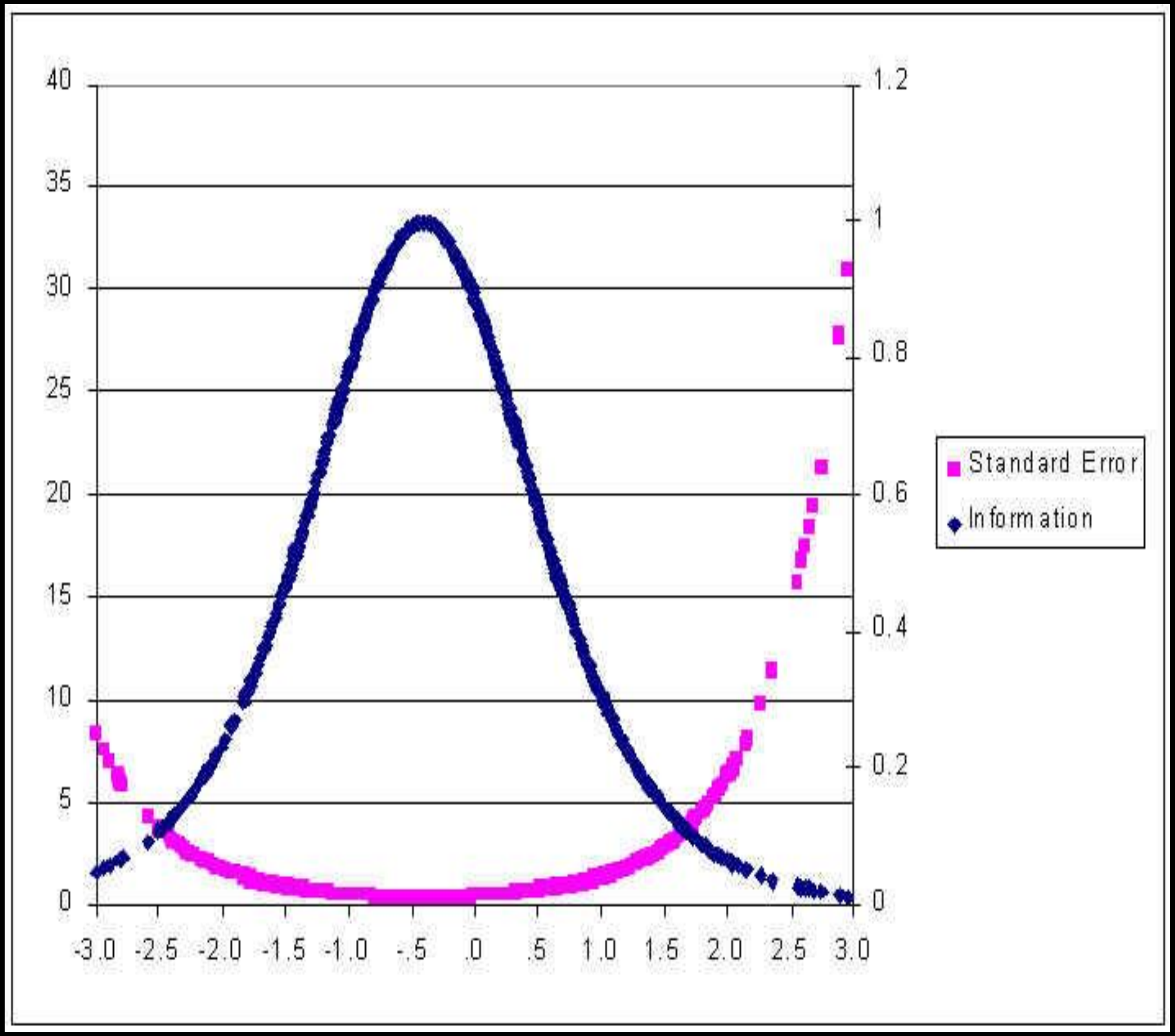
Item Response Theory



Item Response Theory

- Advancements from IRT models
  - More information on item level
    - **Standard Error of Measurement (CTT)**: a single measure for the whole test.
    - **Item and Test Information Function:** inversely related to error, again a function of θ.

Item Response Theory



Item Response Theory

- Advancements from IRT models
  - More information on item level
    - Estimates for a range of ability traits (theta)
    - Difficulty (p-value vs *b*)
    - Discrimination (bisserial correlation vs *a*)
    - Information (SEM vs IIF)
  - Consequence: **sample independence**

Item Response Theory

- Unidimensionality assumption
  - Sample independence is only true (and relevant) if the assessed trait **explains enough response variance**.
  - What to do with inherently multidimensional constructs?
    - Personality
    - Executive Functions
    - Intelligence

Item Response Theory

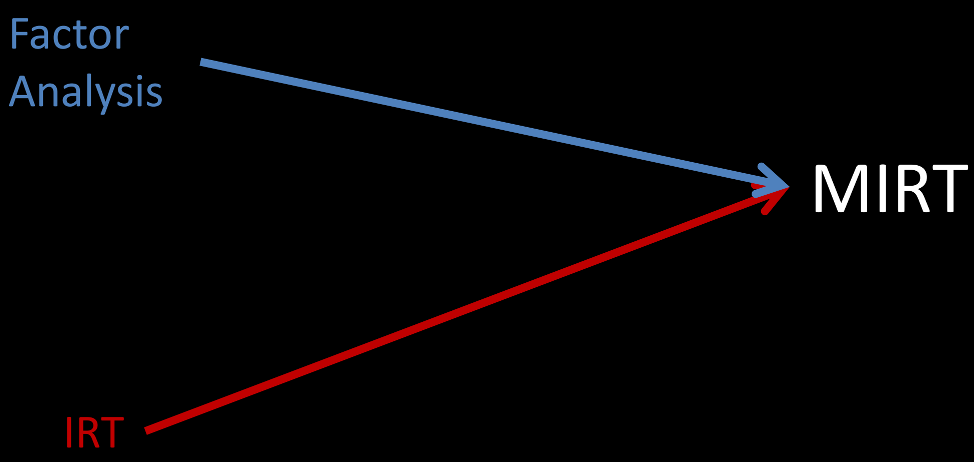
- What to do with inherently multidimensional constructs?
    - Separate the test into **unidimensional subtests**?

  – Issues:
    - Each subtest has to be considered valid on its own, which results in very long and tiresome instruments.
    - What to do with items that load in more than one dimension?

  – Thats when MIRT comes in!

Multidimensional Item Response Theory

- Historical background
  - Earliest MIRT models date from 1970s
    - Reckase (1972), Mulaik (1972), Sympson (1978) and Whitely (1980).



- Construct dimensionality
- Item loadings
- Item parameters
- Probability of scores as a result of the interaction between items and person abilities

Multidimensional Item Response Theory

- Example (Reckase, 2009, p.80)
  - Math items with two factors:
    - Arithmetic problem solving (θ_1)
    - Algebraic symbol manipulation (θ_2)

1. A survey asked a sample of people which of two products they preferred. 50% of the people said they preferred Product A best, 30% said they preferred Product B, and 20% were undecided. If 1,000 people preferred Product A, how many people were undecided?

   A. 200
   B. 400
   C. 800
   D. 1,200
   E. 2,000

Multidimensional Item Response Theory

- Observed data:

Table 1. Proportions of correct responses to Item 1 for 4,114 participants (Reckase, 2009, p.81)

Midpoints in θ_1	**Midpoints in θ_2**							
	-1.75	**-1.25**	**-0.75**	**-0.25**	**0.25**	**0.75**	**1.25**	**1.75**
-1.25	0.20		0.09					
-0.75	0.06	0.18	0.39	0.47	0.19	0.67		
-0.25	0.18	0.25	0.30	0.45	0.54	0.50	0.61	0.82
0.25	0.19	0.40	0.39	0.53	0.45	0.46	0.77	0.57
0.75	0.24	0.34	0.49	0.53	0.50	0.65	0.76	0.71
1.25	0.30	0.35	0.54	0.55	0.47	0.63	0.78	0.55
1.75	0.51	0.55	0.57	0.62	0.60	0.71	0.71	0.65

Multidimensional Item Response Theory

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Proportions in θ_2	0.24	0.34	0.40	0.52	0.46	0.60	0.73	0.66

Multidimensional Item Response Theory

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-1.25	0.20		0.09						0.15
-0.75	0.06	0.18	0.39	0.47	0.19	0.67			0.33
-0.25	0.18	0.25	0.30	0.45	0.54	0.50	0.61	0.82	0.46
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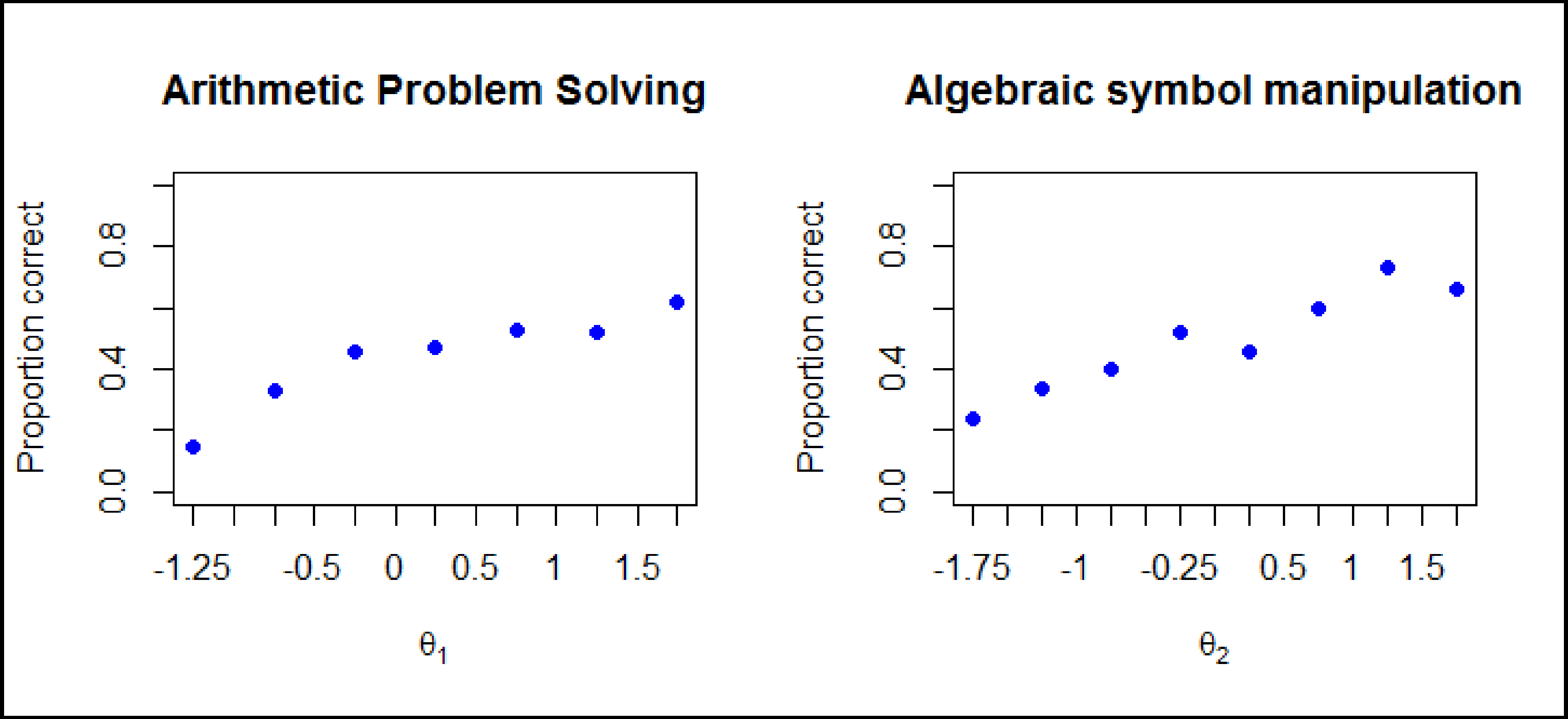
Unidimensional Item Response Theory

- If we assumed the data to be Unidimensional
  - We could only model θ_1 or θ_2

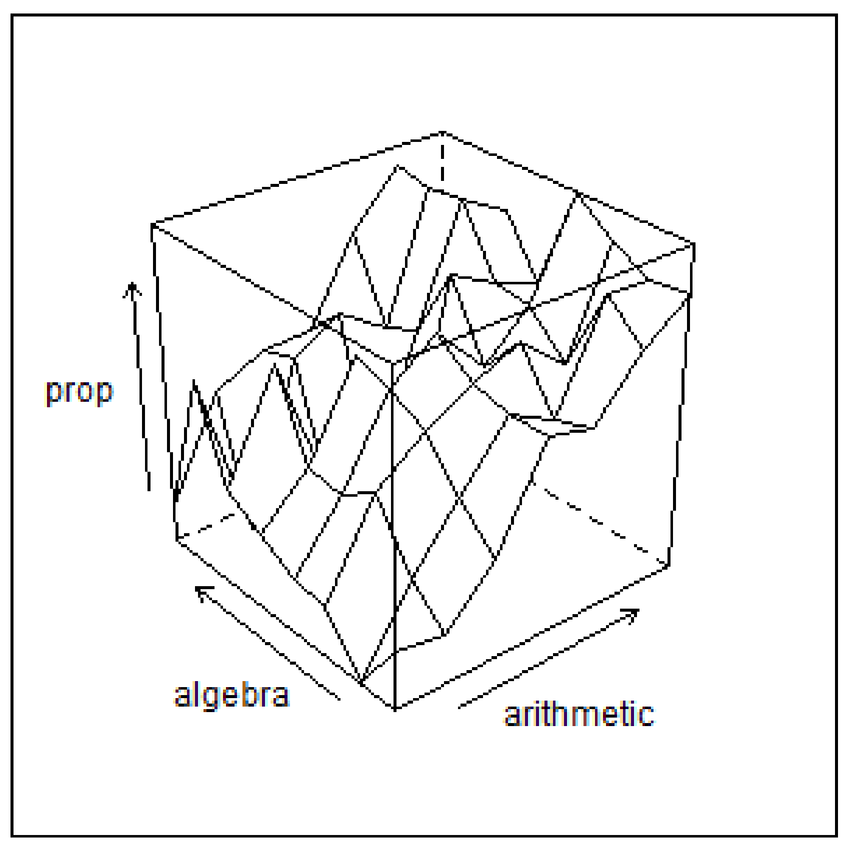


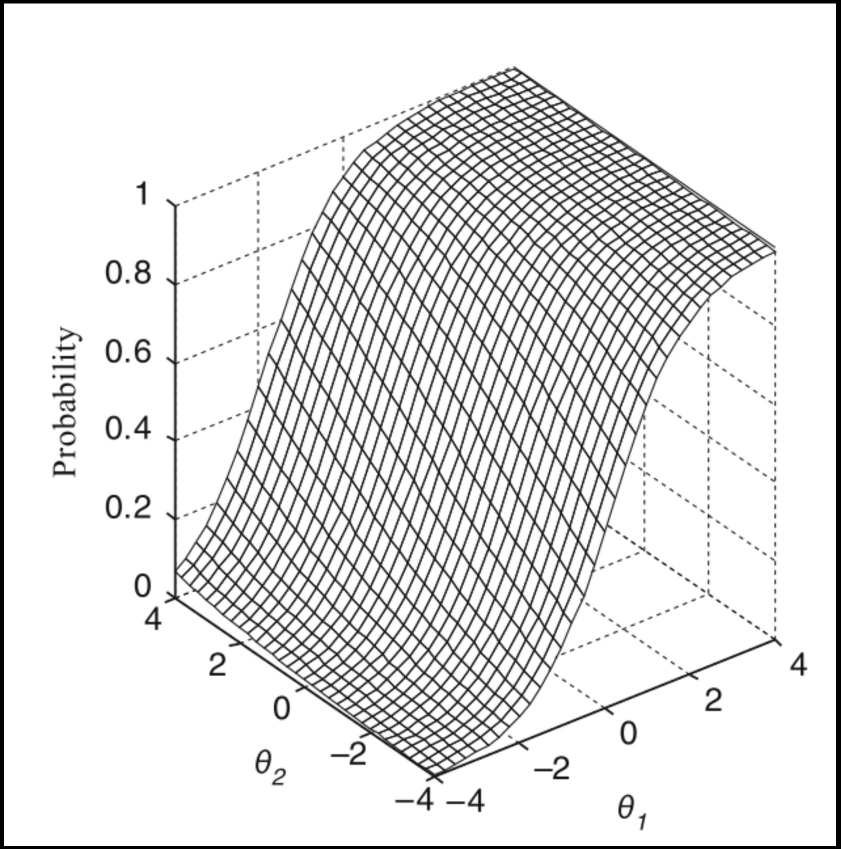
Multidimensional Item Response Theory

- Displays of MIRT models
  - Surfaces
  - Examples in two dimensions only
    - Two coordinate axes are necessary to describe the ability level.
      - θ_1 and θ_2
    - A third axis to represent the proportion of correct responses or the probability $P(1|\theta_{j1}, \theta_{j2})$.

Multidimensional Item Response Theory

- Observed data and Modelled surface





Multidimensional Item Response Theory

- Assumptions
  - Monotonicity assumption
  - Local Independence assumption
    - The term “local” understood as: in that position of the θ_m space

- Major groups of models
  - Compensatory
  - Non-compensatory (partially compensatory)

Multidimensional Item Response Theory

- Models
  - Compensatory
    - Dimensions combine linearly to produce probability of scoring (endorsing) the item
    - High scores in a dimension can *compensate* lower scores in other dimensions.
    - By the time of Reckase's book, only compensatory models were defined for polytomous items.
    - Simpler and most common on literature (Reckase, 2009).

Multidimensional Item Response Theory

- Models
  - Non-compensatory
    - Also called *partially compensatory* models
    - Each dimension is treated separately and the final estimated probability is the product of the individual probabilities
    - Hence, results are a nonlinear combination of the thetas.
      - A very low probability will never be compensated by a higher ability level on another factor.

Multidimensional Item Response Theory

- MIRT Models (dichotomous data):
  - Compensatory models
    - Multidimensional extension of the 2PLM

$$P\left(U_{ij} = 1 \middle| \theta_j, a_i, b_i\right) = \frac{e^{a_i(\theta_j - b_i)}}{1 + e^{a_i(\theta_j - b_i)}}$$

Unidimensional 2PLM

Where, $i = 1, 2, \ldots, \#items$
$j = 1, 2, \ldots, \#\ participants$
$U_{ij} = score\ of\ person\ j\ on\ item\ i$
$a_i = discrimination\ for\ item\ i$
$b_i = difficulty\ for\ item\ i$
$\theta_j = location\ of\ subject\ j\ on\ \theta$

Multidimensional Item Response Theory

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$b_i = difficulty\ for\ item\ i$

$\theta_j = location\ of\ subject\ j\ on\ \theta$

$$P\left(U_{ij}=1\middle|\boldsymbol{\theta}_j,\boldsymbol{a}_i,d_i\right)=\frac{e^{\boldsymbol{a}_i\boldsymbol{\theta}_j'+d_i}}{1+e^{\boldsymbol{a}_i\boldsymbol{\theta}_j'+d_i}}$$

Multidimensional 2PLM

Multidimensional Item Response Theory

- Multidimensional extension of the 2PL

$$P\left(U_{ij} = 1 \middle| \boldsymbol{\theta}_j, \boldsymbol{a}_i, d_i \right) = \frac{e^{\boldsymbol{a}_i \boldsymbol{\theta}_j' + d_i}}{1 + e^{\boldsymbol{a}_i \boldsymbol{\theta}_j' + d_i}}$$

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Both $\boldsymbol{\theta}$ and $\boldsymbol{a}$ are now
1 x m vectors.
$m = \#\, dimensions$

$$\boldsymbol{a}_i = [\, a_{i1}\ a_{i2}\ \ldots\ a_{im}\,]$$
$$\boldsymbol{\theta}_j = [\, \theta_{j1}\ \theta_{j2}\ \ldots\ \theta_{jm}\,]$$

A discrimination statistic (MDISC or A_i) that summarizes the $\boldsymbol{a}_i$ vector is available:

$$A_i = \sqrt{\sum_{k=1}^{m} a_{ik}^2}.$$

Multidimensional Item Response Theory

- Multidimensional extension of the 2PL

$$P\left(U_{ij}=1\middle|\boldsymbol{\theta}_j,\boldsymbol{a}_i,d_i\right)=\frac{e^{\boldsymbol{a}_i\boldsymbol{\theta}_j'+d_i}}{1+e^{\boldsymbol{a}_i\boldsymbol{\theta}_j'+d_i}}$$

This parameter is defined as an intercept, or a location parameter.

Note that in this first generalisation, even though *d* originates from the product of *a* and *b*, it is not a vector but a **scalar**.

Derivation of d:

$$a(\theta - b)$$

$$a\theta - ab$$

Let,
$$d = -ab$$

$$a\theta + d$$

Multidimensional Item Response Theory

- Multidimensional extension of the 2PL

$$P\left(U_{ij} = 1 \middle| \boldsymbol{\theta}_j, \boldsymbol{a}_i, d_i\right) = \frac{e^{\boldsymbol{a}_i \boldsymbol{\theta}_j' + d_i}}{1 + e^{\boldsymbol{a}_i \boldsymbol{\theta}_j' + d_i}}$$

Derivation of d:

$a(\theta - b)$

$a\theta - ab$

Let,
$d = -ab$

$a\theta + d$

If $-d$ is divided by an element of a_i, we obtain a measure of difficulty associated with that dimension.

A summary "difficulty" (MDIFF or b) for the whole item is obtained by:

$$b = \frac{-d}{\sqrt{\mathbf{aa}'}} = \frac{-d}{\sqrt{\sum_{v=1}^{m} a_v^2}}.$$

Multidimensional Item Response Theory

- Multidimensional extension of the 2PL

$$P\left(U_{ij} = 1 \middle| \boldsymbol{\theta}_j, \boldsymbol{a}_i, d_i\right) = \frac{e^{\boldsymbol{a}_i \boldsymbol{\theta}_j' + d_i}}{1 + e^{\boldsymbol{a}_i \boldsymbol{\theta}_j' + d_i}}$$

The exponent is a linear combination of discrimination (a_i) and θ values.

$$\mathbf{a_i}\boldsymbol{\theta}_\mathbf{j}' + d_i = a_{i1}\theta_{j1} + a_{i2}\theta_{j2} + \cdots + a_{im}\theta_{jm} + d_i = \sum_{\ell=1}^{m} a_{i\ell}\theta_{j\ell} + d_i\,.$$

1st Dimension — 2nd Dimension — mth Dimension

The higher the result of the sum, the higher the probability of correct response. Each a_i works as a weight for the total sum. Hence, the more discriminative the item is on a particular dimension, the more influence it should have on the outcome.

Multidimensional Item Response Theory

- Compensatory feature

$$\mathbf{a_i\theta_j}' + d_i = a_{i1}\theta_{j1} + a_{i2}\theta_{j2} + \cdots + a_{im}\theta_{jm} + d_i = \sum_{\ell=1}^{m} a_{i\ell}\theta_{j\ell} + d_i.$$

1st Dimension 2nd Dimension mth Dimension

  – Let the exponent be equal to a given value k

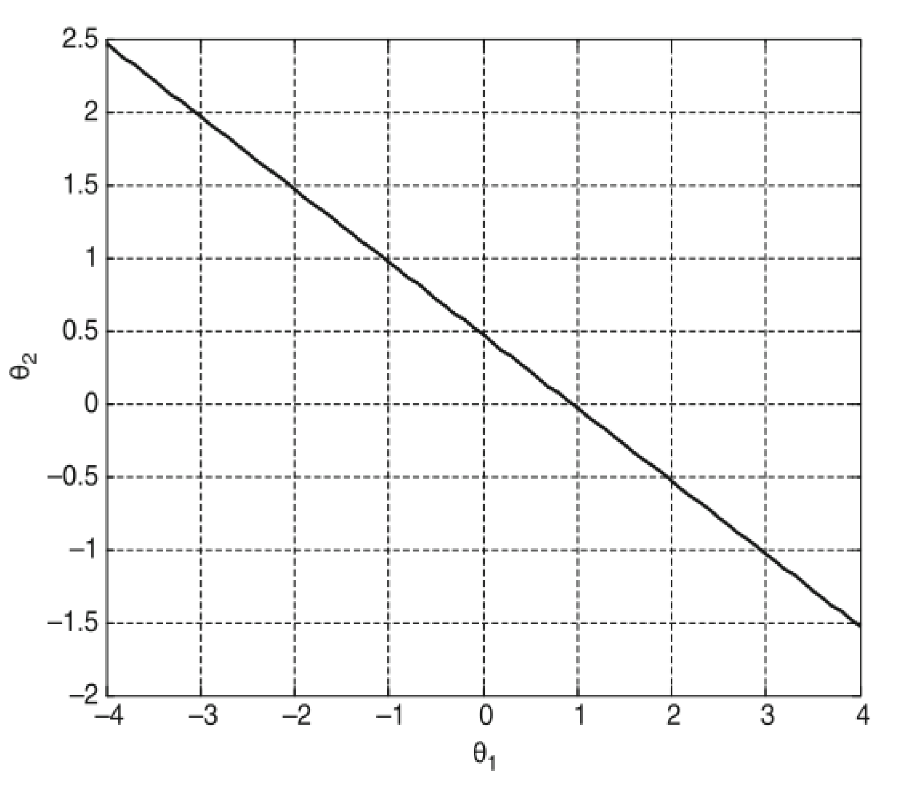
    - Then, all $\boldsymbol{\theta}$ vectors that satisfy $k = \boldsymbol{a_i\theta_j'} + d_i$ fall in a straight line.

Multidimensional Item Response Theory

- Compensatory feature
  - $k = \boldsymbol{a}_i \boldsymbol{\theta}_j' + d$
  - For example, let $k = 0$
    - Let item *i* be defined in two dimensions and have parameters $a_i = [.75 \ 1.5]$ and $d_i = -.7$
    - The exponent then becomes:
      - $k = .75\theta_1 + 1.5\theta_2 - .7 = 0$

Multidimensional Item Response Theory

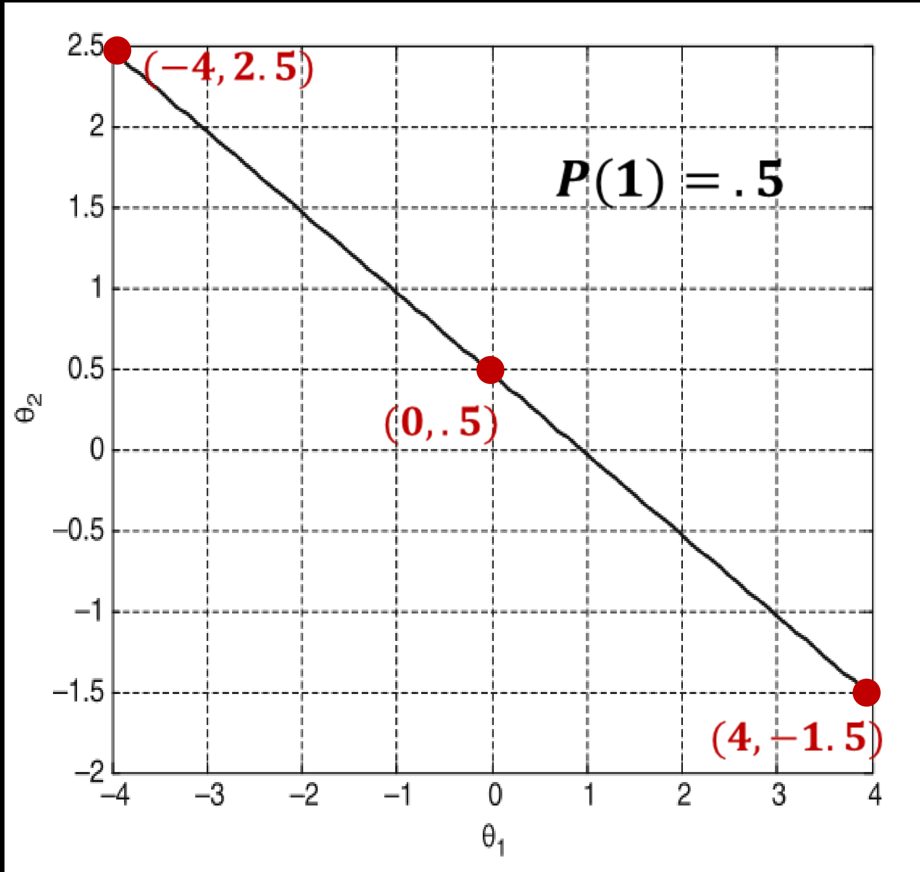
- Compensatory feature
  - Solving for θ_2
    - $\theta_2 = -.5\theta_1 + \frac{.7}{1.5}$



– Plot of theta vectors that yield exponents of $k = 0$ for a test item with parameters $a_1 = .75$, $a_2 = 1.5$ and $d = -.7$

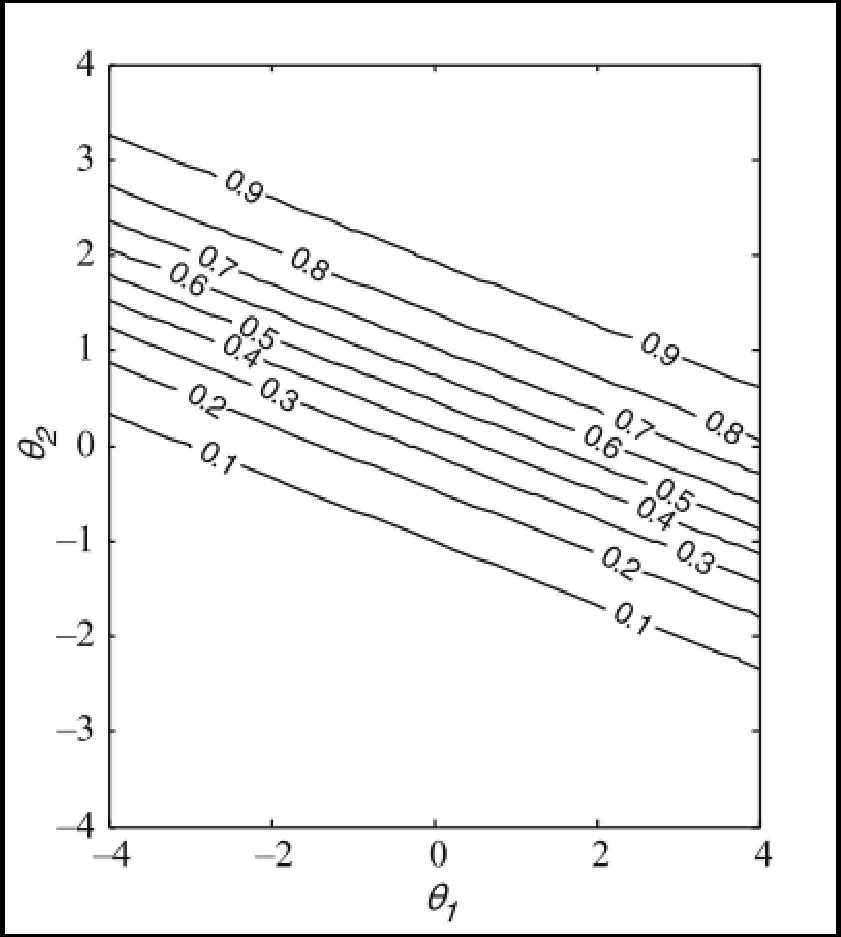
Multidimensional Item Response Theory

- Compensatory feature
  - $\mathrm{P}(1) = \frac{e^0}{1+e^0} = \frac{1}{2} = 0.5$
    - $\boldsymbol{\theta} = [0\ .5]$
    - $\boldsymbol{\theta} = [-4\ 2.5]$
    - $\boldsymbol{\theta} = [4\ -1.5]$
  - Different combinations of θ_1 and θ_2 yield the same probability of correct response



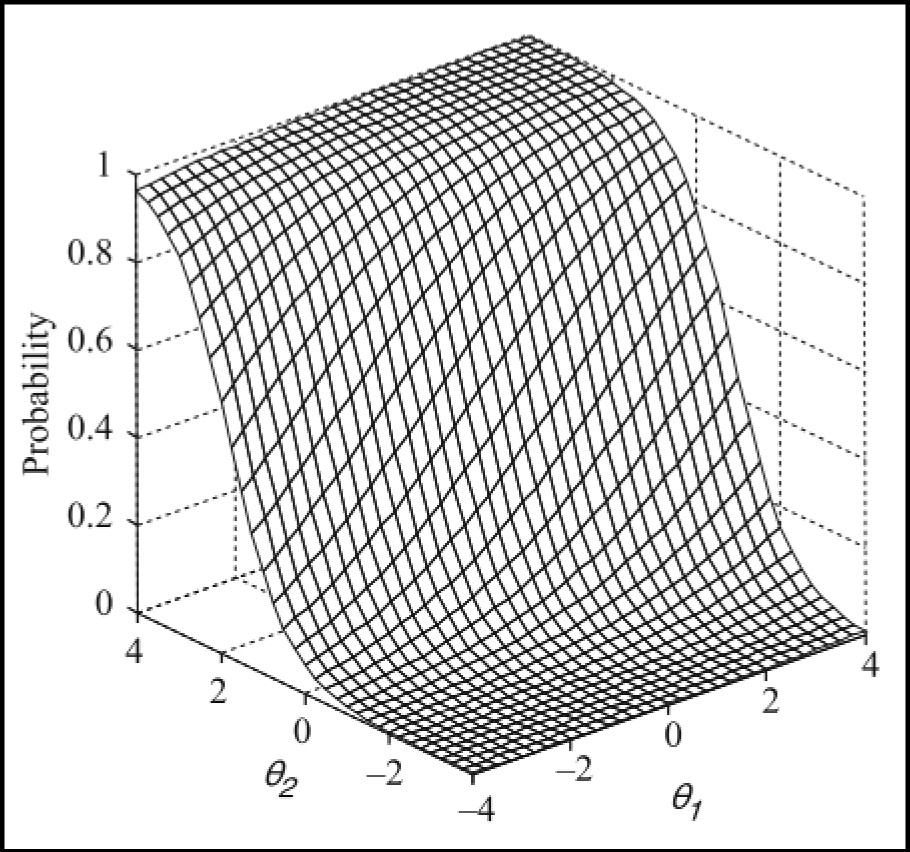
Multidimensional Item Response Theory

- Compensatory Feature
  - The same can be repeated for several different probability lines
    - Probability contours
  - This item has parameters $a_1 = .5$, $a_2 = 1.5$ and $d = -.7$



Multidimensional Item Response Theory

- Item Response Surface
  - IRS



- And now for all possible probability values.
  - Item parameters: $a_1 = .5$, $a_2 = 1.5$ and $d = -.7$

Multidimensional Item Response Theory

- Both plots:

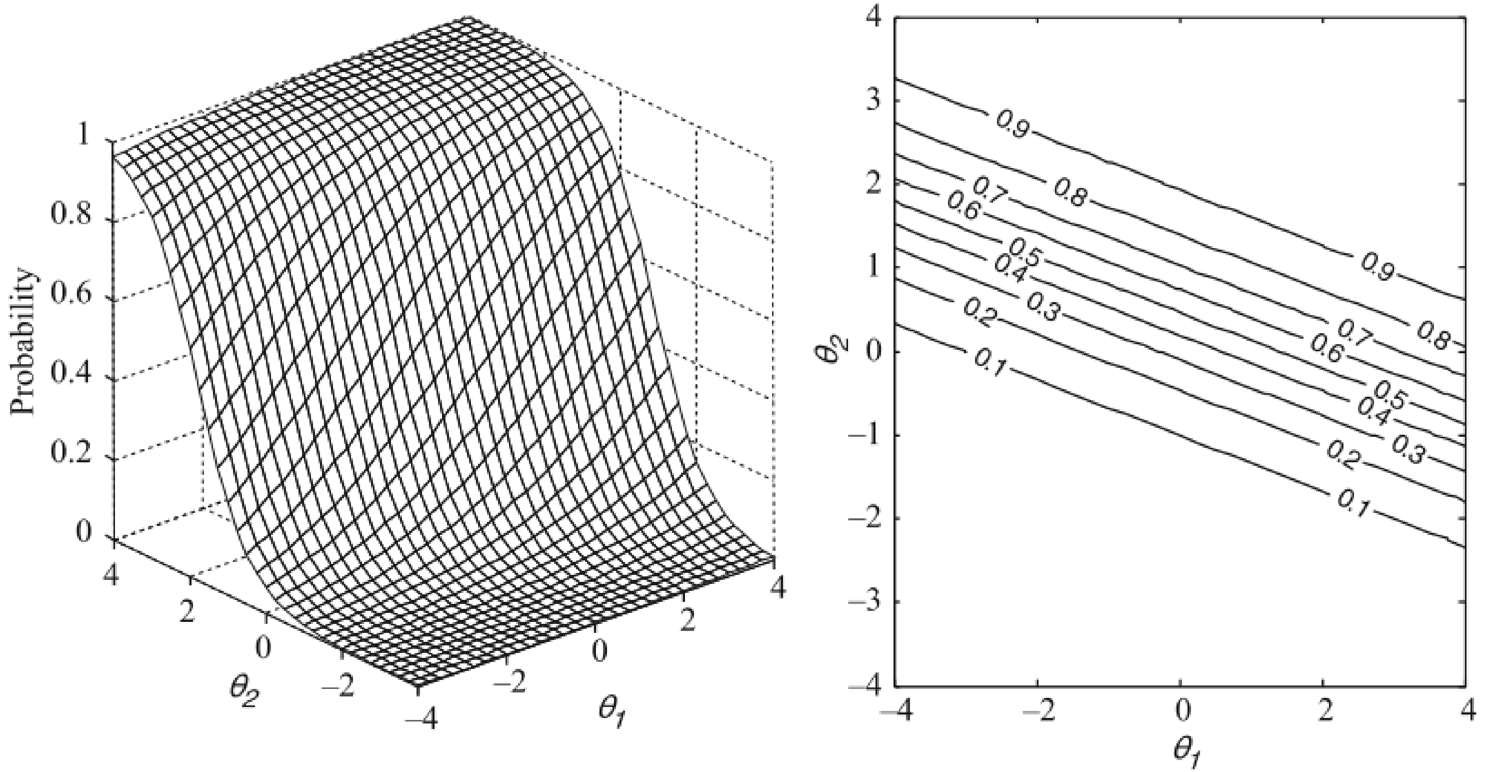


Fig. 4.2 Surface plot and contour plot for probability of correct response for an item with $a_1 = .5$, $a_2 = 1.5$, $d = -.7$

Multidimensional Item Response Theory

- A different example. What has changed?

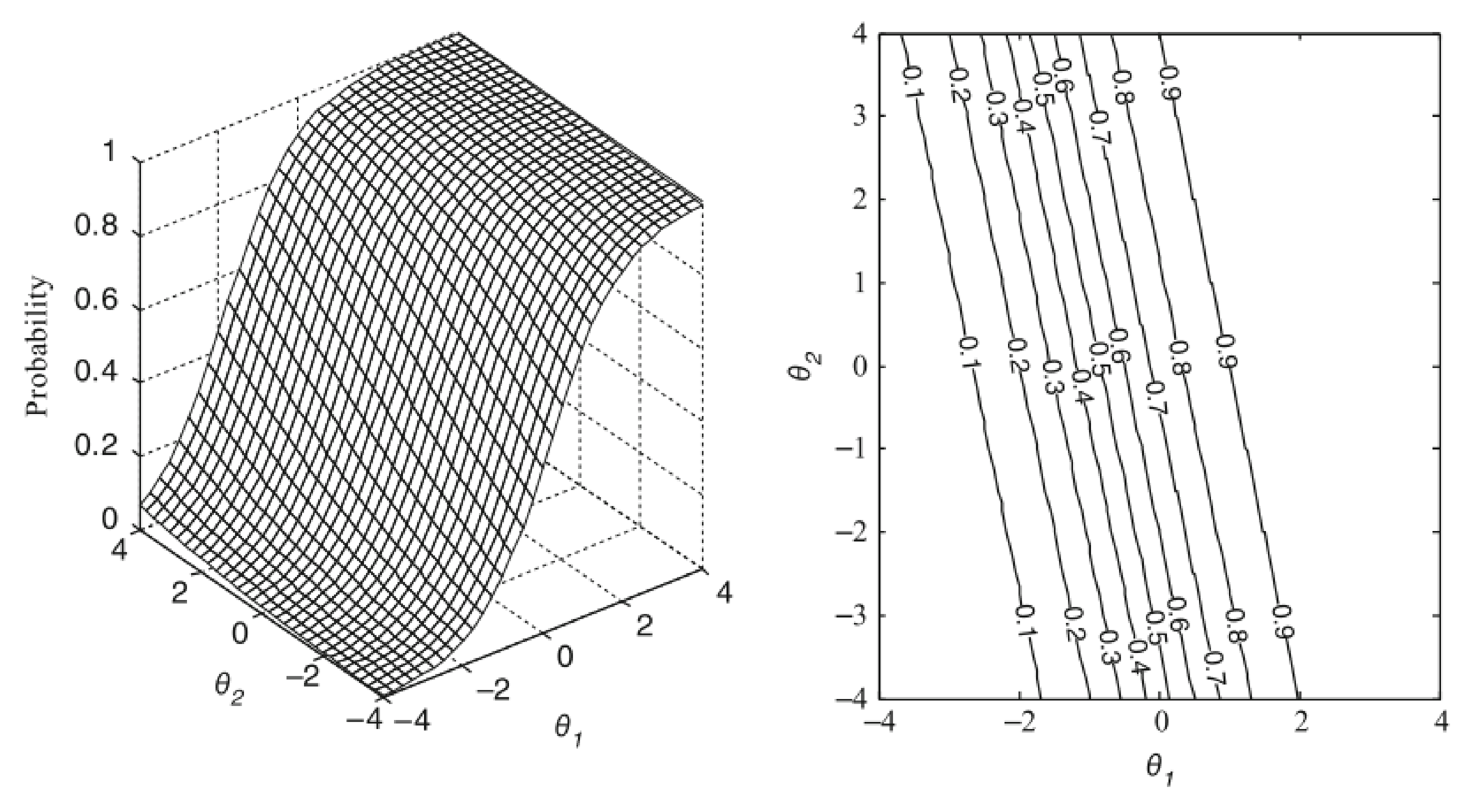


Fig. 4.3 Surface plot and contour plot for the probability of correct response for an item with $a_1 = 1.2, a_2 = .3, d = 1$

Multidimensional Item Response Theory

- A quick glance at the M3PLM
  - The lower asymptote
  - $P\left(U_{ij} = 1 \middle| \boldsymbol{\theta}_j, \boldsymbol{a}_i, c_i, d_i\right) = c_i + (1 - c_i)\dfrac{e^{\boldsymbol{a}_i\boldsymbol{\theta}_j' + d_i}}{1 + e^{\boldsymbol{a}_i\boldsymbol{\theta}_j' + d_i}}$

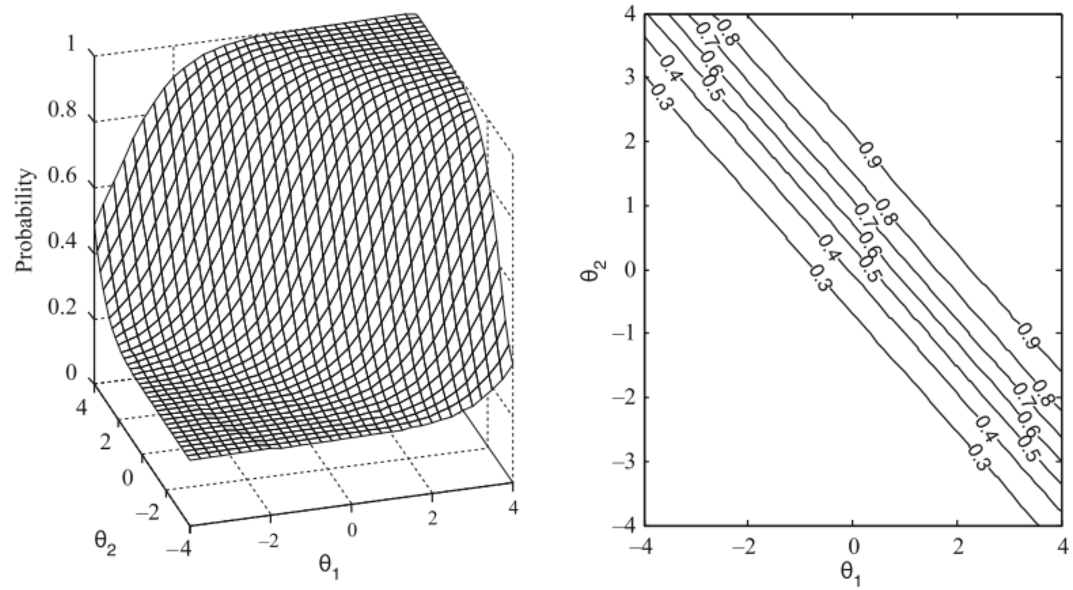


Fig. 4.5 Surface plot and contour plot for probability of correct response for an item with $a_1 = 1.3$, $a_2 = 1.4$, $d = -1$, $c = .2$

Multidimensional Item Response Theory

- Non-compensatory models
  - The rationale behind:
    - Suppose the introductory item measured, instead:
      - Arithmetic problem solving (θ_1)
      - Reading skill (θ_2)
    - A person with very low reading skills attempts the example item. Even with extremely high mathematic skills, an individual would not obtain success on the item.

Multidimensional Item Response Theory

- Non-compensatory models
  - Mathematical model:

$$P(U_{ij} = 1|\boldsymbol{\theta}_{\mathbf{j}}, \mathbf{a}_{\mathbf{i}}, \mathbf{b}_{\mathbf{i}}, c_i) = c_i + (1 - c_i)\left(\prod_{\ell=1}^{m} \frac{e^{1.7a_{i\ell}(\theta_{j\ell} - b_{i\ell})}}{1 + e^{1.7a_{i\ell}(\theta_{j\ell} - b_{i\ell})}}\right)$$

  - Sympson (1978).

Multidimensional Item Response Theory

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  - The b_i also becomes a $1\ x\ m$ vector
  - Each unidimensional probability is calculated and their overall product is the estimated probability of scoring the item.
  - Model was originally devised with a lower asymptote

Multidimensional Item Response Theory

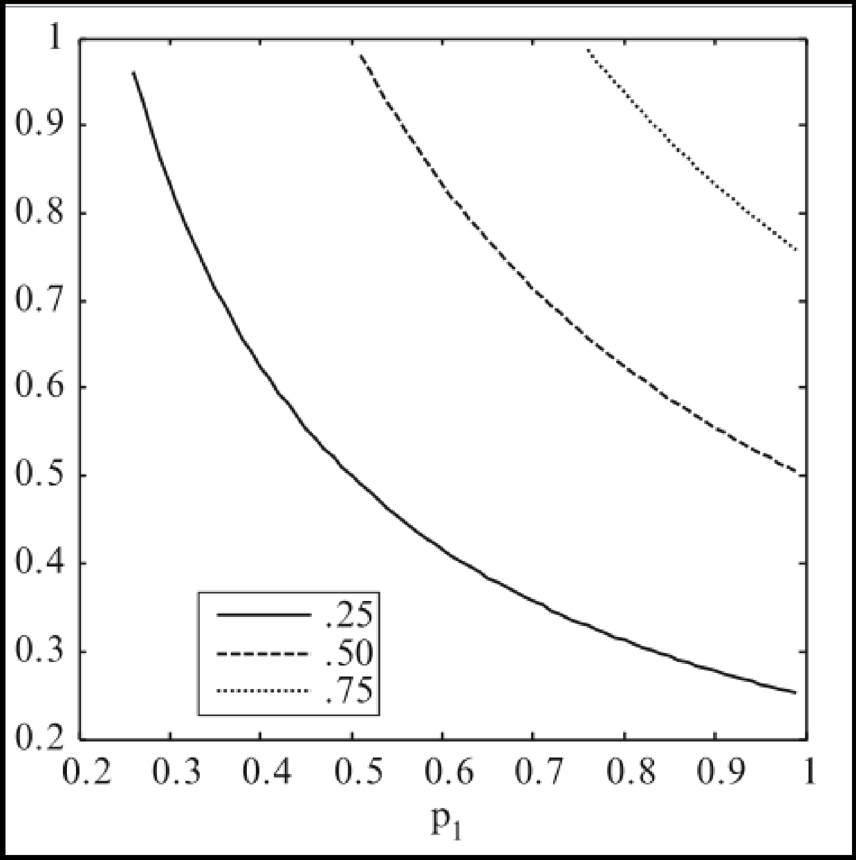
- Non-compensatory models
  - Probability contours
    - For the case where $c = 0$ Consider this simpler model:

$$k = \prod_{\ell=1}^{m} p_\ell$$

    - p_ℓ is the probability of scoring correctly in each dimension (ℓ).
    - For the simpler two dimensional case: $k = p_1 p_2$
      - This yields the following plot:

Multidimensional Item Response Theory

- Non-compensatory models
  - Probability contours (for $k = .25, .50\ and\ .75$)



Mathematicaly, these are hyperbolas

Not yet a function of $\boldsymbol{\theta_j}$

Multidimensional Item Response Theory

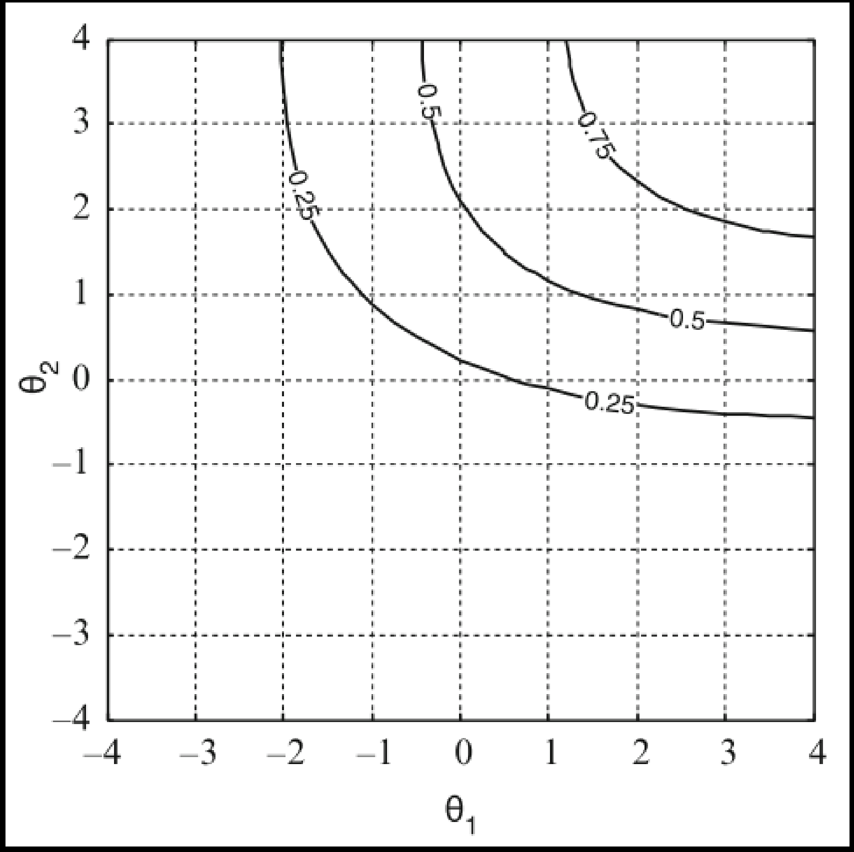
- Non-compensatory models
  - Probability contours (for $k = .25, .50 \; and \; .75$)

Now as a function of $\boldsymbol{\theta_j}$

Interesting feature: the .5 probability curve asymptotes to the values of b_i

If, say, $\theta_1 = b_{i1}$ then the unidimensional $P(1|\theta_1, b_i) = .5$.
Since you are multiplying probabilities [0,1], this becomes the highest possible probability value, for a subject with this specific θ_1,

$\boldsymbol{c_i = 0}, \quad \boldsymbol{a_{i1} = .7}, \quad \boldsymbol{a_{i2} = 1.1}, \quad \boldsymbol{b_{i1} = -.5 \; and \; b_{i2} = .5}$



Multidimensional Item Response Theory

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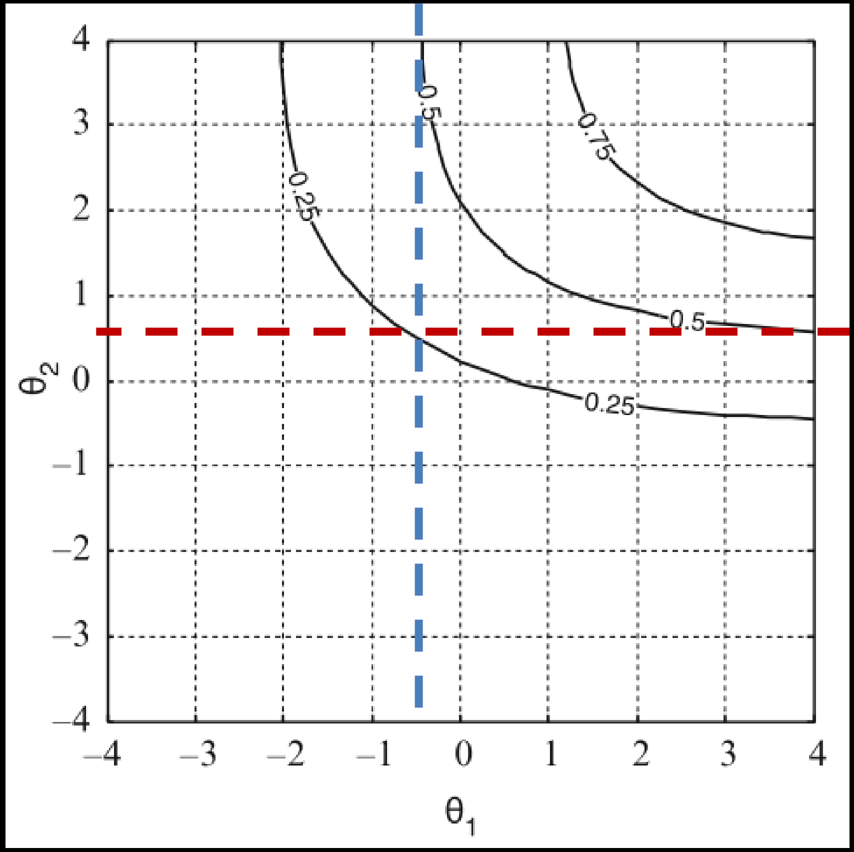

Multidimensional Item Response Theory

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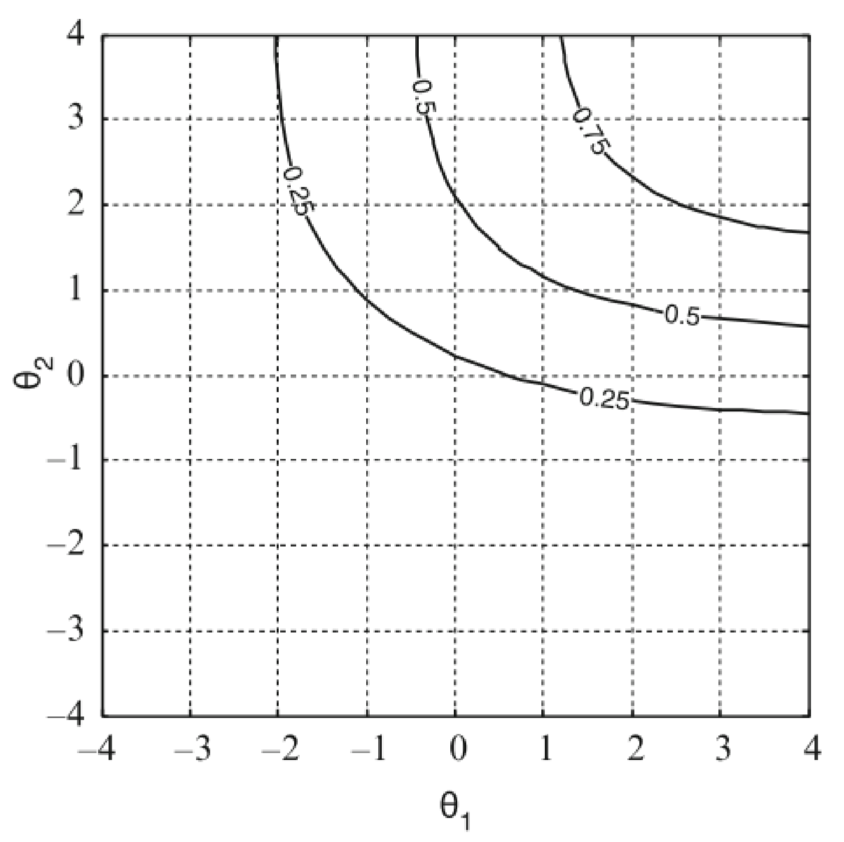
Now as a function of $\boldsymbol{\theta_j}$

Interesting feature: the .5 probability curve asymptotes to the values of b_i

The probability of correct response for an item that follows this model can never be greater than the probability for the component with the lowest probability

$$\boldsymbol{c_i = 0, \quad a_{i1} = .7, \quad a_{i2} = 1.1,}$$
$$\boldsymbol{b_{i1} = -.5 \; and \; b_{i2} = .5}$$



Multidimensional Item Response Theory

- Non-compensatory models
  - Item Response Surface (IRS):

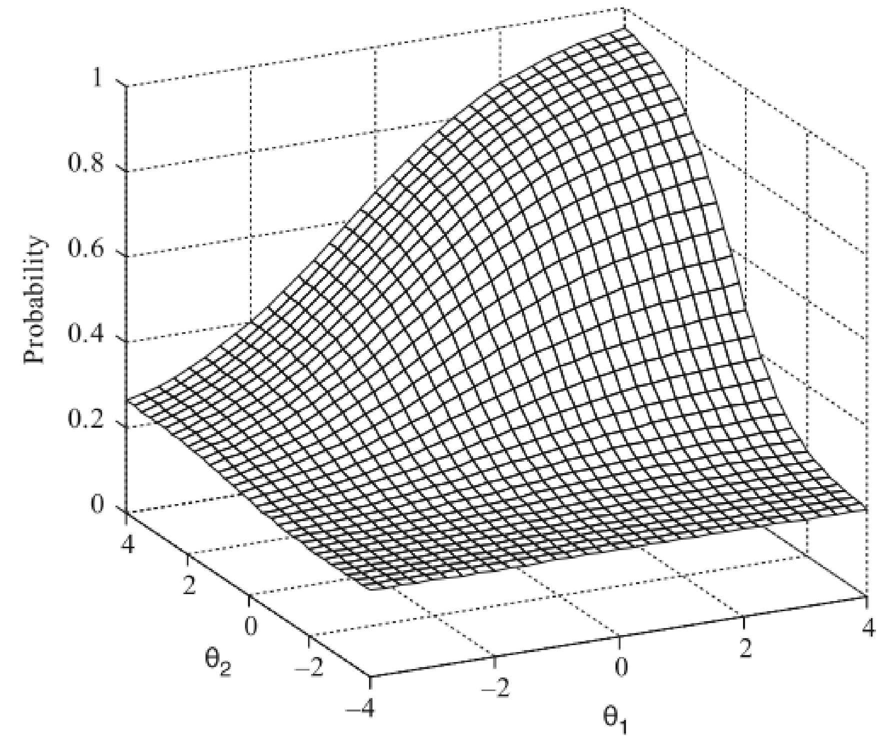


Fig. 4.9 Item response surface for the partially compensatory model when $a_1 = .7$, $a_2 = 1.1$, $b_1 = -.5$, $b_2 = .5$, and $c = .2$

Multidimensional Item Response Theory

- Non-compensatory models
  - Item parameters as number of dimensions increase.
    - In UIRT: if $\theta_j = b_i$, then $P(1) = .5$
    - In Compensatory MIRT: if $\boldsymbol{\theta}_j$ is the 0-vector and $d = 0$, then $P(1) = .5$
    - For Partially Compensatory models this is not true
      - For $m = 2$, if $\boldsymbol{\theta}_j = \boldsymbol{b}_i$, then $P(1) = .25$
      - For $m = 3$, if $\boldsymbol{\theta}_j = \boldsymbol{b}_i$, then $P(1) = .125$
      - For any m, if $\boldsymbol{\theta}_j = \boldsymbol{b}_i$, then $P(1) = .5^m$

Multidimensional Item Response Theory

- Non-compensatory models
  - Item parameters as number of dimensions increase.
    - $P(1) = .5$ for the case where $\boldsymbol{\theta}_j = 0$, all $\boldsymbol{a}_i = .588$ (because of D constant) and all $\boldsymbol{b}_i$s are equal.

Number of dimensions	b-parameter
1	0
2	−.88
3	−1.35
4	−1.66
5	−1.91
6	−2.10

Multidimensional Item Response Theory

- When to use each type of models?
  - Ultimately, the fit to the data will define
  - For positively correlated dimensions expect little difference.
  - According to Reckase (2009) there are few studies that compare fit from both models.

Multidimensional Item Response Theory

- Multidimensional Item Information Function:
  - From UIRT: item information relates to the slope parameter.
    - The bigger the slope the higher the information function.
  - The same happens in MIRT
    - Issue: at each point of the IRS a different slope exists depending on the chosen direction.

Multidimensional Item Response Theory

- Multidimensional Item Information Function:
  - One solution: the Item Information Surface is displayed across different angles.

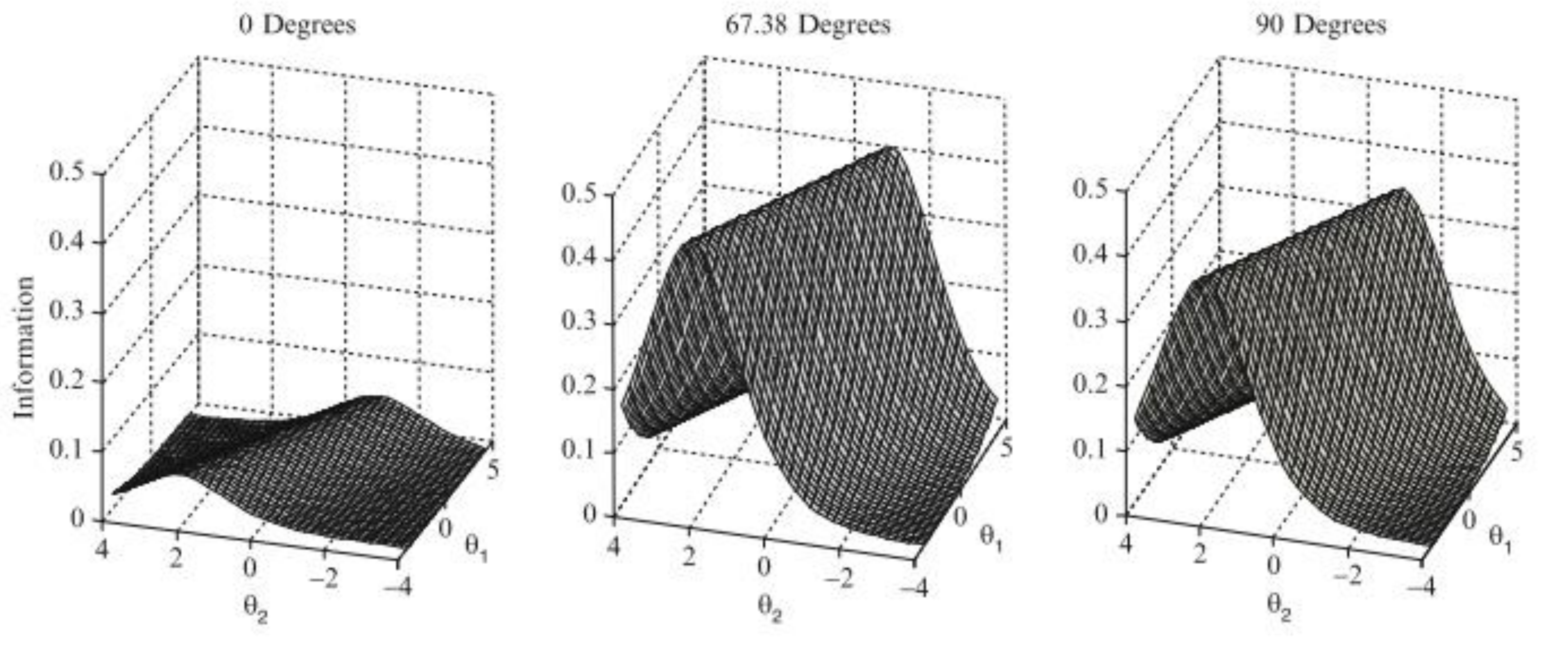


Fig. 5.6 Information surfaces for a M2PL test item with $a_1 = .5$, $a_2 = 1.2$, and $d = -.6$ in three directions

Multidimensional Item Response Theory

- Multidimensional Item Information Function:
  - A grid of points in the θ space, where the information is depicted in different angles by small intervals.

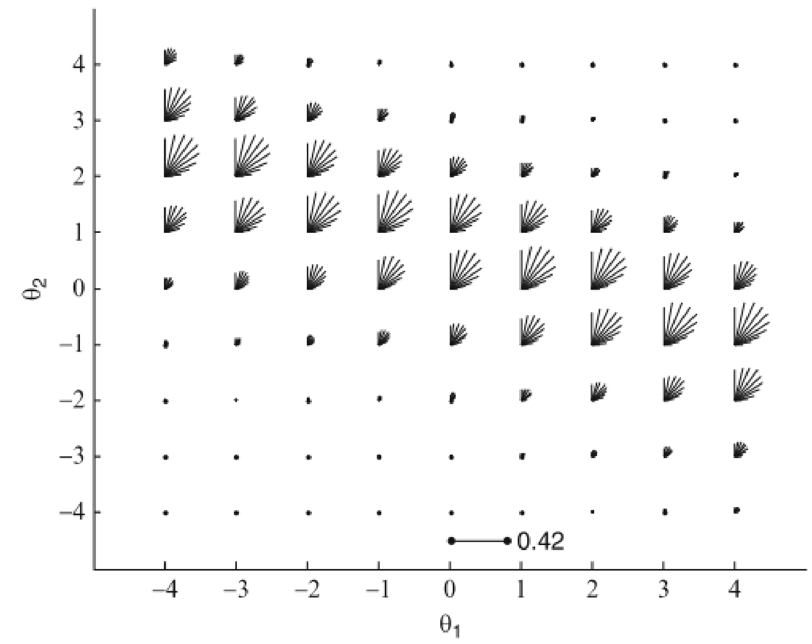


Fig. 5.7 Information for a M2PL test item with $a_1 = .5$, $a_2 = 1.2$, and $d = -.6$ at equally spaced points in the $\boldsymbol{\theta}$-space for angles from 0° to 90° at 10° intervals

Multidimensional Item Response Theory

- Applications:
  - Test length reduction with MCAT
    - By allowing a single item to provide information for more than one dimension, test length can be reduced significantly. Specially using MCAT.
  - Differential Item Functioning
    - Identifying the extent to which the underlying element is causing unexpected invariance.
  - Progressess on fields such as abnormal response patterns.
    - An underlying trait could be modelled

Thank You!

- For further questions please e-mail me at: victorduran89@gmail.com

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